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NUMBER THEORY.

204. Proposed by E. T. BELL, New York City.

Show that a necessary and sufficient condition that $6n + 1$ be a prime number is that no one of the quantities $(3m - r)/(2r + 1)$ for $r = 1, 2, 3, \cdots n - 1$ be an integer. Similarly for $6n - 1$, the quantities being $(3n - r)/(2r - 1)$ for $r = 2, 3, 4, \cdots n$.

SOLUTION BY ELIJAH SWIFT, Princeton, N. J.

Suppose first that the condition be not satisfied. Then we have for some $r < n$, $(3n - r)/(2r + 1) =$ an integer, say α, which is equivalent to the equation $6n + 1 = (2r + 1)2\alpha + 2r + 1$. Hence, $6n + 1$ has the factor $2r + 1$ and is not prime, so that the condition is necessary.

Again suppose that $6n + 1$ is not prime. We shall see that the given condition is not satisfied. In this case the odd number $6n + 1$ has two odd factors, which we will call $2r + 1$ and $2\alpha + 1$. Each must be as large as 5 since 3 is evidently not a factor of $6n + 1$. If $2\alpha + 1 \geqq 5$, $2r + 1 \leqq (6n + 1)/5 = n + (n + 1)/5$. But this is less than $2n - 1$ if $n \geqq 2$, and if $n = 1$, $6n + 1$ is prime. Hence $r \leqq n - 1$. Reversing the algebraic work above we infer from $6n + 1 = (2r + 1)(2\alpha + 1)$ the equation $(3n - r)/(2r + 1) = \alpha$. Hence the condition is sufficient.

The proof of the second half of the theorem is similar mutatis mutandis.

206. Proposed by R. D. CARMICHAEL, Indiana University.

Prove that the sum of the sixth powers of two integers cannot be the square of an integer.

SOLUTION BY ELIJAH SWIFT, Princeton, N. J.

We are required to show that the equation $a^6 + b^6 = c^2$, where a, b, c are integers, is impossible. We may suppose that a, b, and c are prime to each other, as any common factor may be divided out. Now all integral solutions of the equation $x^2 + y^2 = z^2$, where x, y, z are prime to each other are given by the formulas $x = 2mn$, $y = m^2 - n^2$, $z = m^2 + n^2$, where m and n are integers prime to each other, and one is even and one odd. We must, then, have $a^3 = 2mn$, $b^3 = m^2 - n^2$. Suppose that m is even and n odd. Since $2mn$ is a cube, and $2m$ and n are prime to each other, each must be a cube, and $2m$ must contain the factor 8, and m the factor 4. Call $2m = 8\alpha^3$, $n = \beta^3$. Substituting these values in the formula for b^3, we have $b^3 = 16\alpha^6 - \beta^6 = (4\alpha^3 - \beta^3)(4\alpha^3 + \beta^3)$. Since $2m$ and n have no common factor, $4\alpha^3$ and β^3 have none, and $4\alpha^3 - \beta^3$ and $4\alpha^3 + \beta^3$ must also be prime to each other. Hence each is a perfect cube, since their product is, and we have $4\alpha^3 + \beta^3 = q^3$, $4\alpha^3 - \beta^3 = p^3$.

But from these two equations follows at once by addition $8\alpha^3 = p^3 + q^3$ or $r^3 = p^3 + q^3$, which is impossible. If n is even and m is odd the same method applies.

This method is general and may be applied to prove that if a, b, c, and n are positive integers (A) $a^{2n} + b^{2n} = c^2$ is possible only if there exist positive integers p, q, and r such that (B) $p^n + q^n = r^n$. If we assume the truth of Fermat's theorem—namely that (B) is impossible if $n > 2$ — we have shown the impossibility of (A) for every value of n except $n = 1$, or $n = 2$. For $n = 1$ (A) is of course possible; its impossibility for $n = 2$ may be shown by this same method. See Bachmann, *Niedere Zahlentheorie,* vol. II, p. 453.

MISCELLANEOUS QUESTIONS.

Edited by R. D. Carmichael.

QUESTIONS.

13. What are the most important of the newer problems that confront teachers of high school mathematics, and why are they important?

REPLY.

9. What is the present state of experience with coördinated courses in high school mathematics? What contribution does this promise to the development of mathematics teaching in high schools? What about the corresponding matters in college mathematics?

I. Remarks by Edith Long, Lincoln, Nebraska.

That there has been and still is an unrest among thoughtful high school teachers of mathematics is manifest. This problem certainly exists: How are we to adapt the courses in algebra and geometry to the experiences of high school students, in order that these may contribute to their broader development in both a cultural and a practical way? On the one hand we have the student who is preparing for the university and on the other hand the student whose school life ends with the high school or who, going on to the university, will not continue the subject of mathematics. The one who would solve this problem must have these two classes in view, and the solution for the one must be the solution for the other since it is only in the largest schools that distinction can be made by having separate classes, even if it were advisable. It is for the high school teacher to work out a simple course which will serve the purpose of college preparation as well as the one now followed and which at the same time will give to the boy or the girl who does not go on a clear and simple notion of the nature and possibilities of mathematics as a whole. It ought to be possible for pupils to leave the high school with other impressions than that this great subject is embodied in two narrow and distinct divisions called algebra, where one works exercises, and geometry, where one demonstrates theorems. Some impressions of tangible and present value should be made to replace the vague feeling of subtle virtue, not to be understood now, but which in future years it is hoped will show its value and repay for long and weary hours spent.

There is no conflict between these two aims. A course that will satisfy the latter purpose will serve the former far better than those we now have. But before such a course can be formulated and put into effect the barriers of prejudice must be broken down. The tandem plan of algebra, geometry, trigonometry, analytical geometry, and calculus, running through the high school, and the first two years of college, should give way to a new course which will unify these sub-